“Telling” can be an effective tool in helping students engage in intellectually demanding argumentation and productive behavior.
With new standards come new teaching opportunities. The Common Core State Standards for Mathematics (CCSSM) emphasizes argumentation as one of its eight mathematical practices across grades and across mathematical topics. Social and mathematical aspects of argumentation are included in the statement that students should “Construct viable arguments and critique the reasoning of others” (CCSSI 2010, p. 6).

Mathematical argumentation is a special kind of discourse, the goal of which is to determine the truth of mathematical statements. Supporting argumentation is complex, and teachers need a variety of methods for doing so. The method we focus on in this article is giving students advice about how to behave as they engage in mathematical argumentation.

Although much has been written on questioning to support mathematical discourse, telling is an important teaching move, too (Chazan and Ball 1999; Parks 2010).

Mathematical argumentation can be viewed as a social activity with three parts, in which students work together to (1) make conjectures, (2) justify the conjectures, and (3) decide whether they are true or false (i.e., conclude) (Erduran, Simon, and Osborne 2004; Sowder and Harel 1998; Zohar and Nemet 2002). Conjecturing is a process of conscious guessing (Lakatos 1976) or pattern finding to create mathematical statements of as-yet undetermined mathematical validity (Harel 2008). Justifying is a process of explicating one’s reasoning to establish the mathematical validity of a conjecture. Concluding is the process of coming to consensus or agreement about the validity of the conjecture and its justification (Shechtman and Knudsen 2009).

Productively engaging students in mathematical argumentation is
one of many challenges in the era of CCSSM. We report on successful episodes of argumentation conducted by teachers and students participating in the Bridging Professional Development project as well as several teacher moves that could help engage students’ argumentation. Teachers Abby, Bernie, Debbie, and Stephanie (pseudonyms) were all middle school teachers whose work is portrayed in a series of vignettes, in which they both tell and show students how to argue.

**MATERIAL SUPPORTS FOR ARGUMENTATION**

In the Bridging project, teachers participated in professional development and received classroom materials that included activity sheets, software, and posters. Teachers used one of two sets of activities. In Patterns of Coordinates, students draw four rectangles on a coordinate grid, label the coordinates of the vertices, look for patterns in the coordinates of these rectangles, make conjectures based on those patterns, and then justify the conjectures. In Triangles: Sides and Angles, students produce a collection of mathematical truths (theorems) that they use to argue that the sum of the measures in a triangle is 180 degrees.

Teach...
new. Mathematics is, by nature, a creative endeavor, as mathematicians uncover new truths with arguments that have yet to be made (de Holton et al. 2001).

How do teachers foster such creativity? They do so, in part, by telling students what is expected of them, by making “creativity” accessible and concrete. Abby, using activities from the Triangles unit, found that students were reluctant to make conjectures that went beyond what they already knew. To foster creative conjecturing, Abby gave a minilecture, weaving in advice related to the How to Do Argumentation poster, which remained displayed in her classroom for the duration of the activities:

Conjectures don’t have to be hugely deep and powerful, but they have to be something that is beyond what we’ve already established. You can say something that, once you think about it, you can change your mind. But still write it down as an immediate conjecture because it might be worthwhile to talk through the justification. I really want to push you guys today. You tend to get a little bit like, “I wrote a conjecture, I’m done.” The point of today is to push you to make as many conjectures as you can. I’d like to ask everyone to list—I’m going to pass out extra paper—at least four conjectures per table; that means at least one per person and one as a group.

Abby gave students details about what it means to “be creative” in making conjectures: go beyond, write down conjectures without judging, and generate many conjectures. She provided extra paper as a concrete representation of her instructions to “make as many conjectures as you can.”

Abby then explained that the class was going to spend fifteen or twenty minutes exploring. Students were to—play, observe, wonder, explore. Really think about these conjectures as you’re playing, and your observations, and your wonderings, and then really explore them, so we can move on from there.

With these directions, Abby provided students with examples of behaviors indicative of creativity—play, observe, wonder, and explore. She encouraged and celebrated such behaviors. Although telling was not the only move Abby made to support students in making more and better conjectures, there is evidence that this minilecture helped. In the small-group work time that followed, students formulated their required four conjectures per group; some groups produced even more.

**Justifying**

Justification occurs in at least three ways in Bridging teachers’ classrooms:

1. Teams or individuals generate conjectures and the whole class justifies them.
2. The whole class together generates and justifies conjectures.
3. Teams or individuals come up with conjectures and justifications, which are shared with the class and brought to conclusion.

Teachers often use the poster in **figure 1** to capture arguments presented to or constructed by the class.

Justifying, at the most basic level, requires students to look for reasons why a conjecture is true or false. The poster makes this explicit for those just beginning to learn the arguing process.

In explaining how to justify, Bernie presented students with three ways to justify: (1) numerically, (2) visually, and (3) geometrically. The words **numeric, visual, and geometric** were written on the white board every day, and Bernie repeated these justifications before engaging in the whole-class sharing of them. Moreover, after some justifications, she also highlighted particular student moves:

What Sandy and Nancy [pseudonyms] just did is a geometric justification. They didn’t use any numbers; they didn’t say anything about it being visual or what it looked like. They used geometry; they used what they knew about the radius of a circle. So we’ll say that it’s true.

Giving justification advice, such as “Look for examples and counterexamples,” can be used at an individual, small-group, or whole-class level. It gives students a place to start.

Looking for examples can lead students to consider whether a conjecture applies to all cases. In Abby’s class, she clarified the use of examples, or cases, with this prompt to help her students make better arguments:

As a group, you need to argue through the conjectures. You need to think about different cases: Is it true in all cases? I really want you to think about using terminology in your justification of linear pairs, exterior angles, congruency; use the math language that we’ve been building. Everyone here, by the end of class today, should have some kind of argument written down.

Abby also stated the expectation that students would make written arguments, a way to ensure that students who participate in collective argumentation are also held accountable at the individual level. Students’ workbooks showed evidence that they made this transition to written, individual argumentation.

Debbie, another teacher, used the Triangles materials to clarify the purpose of both examples and counterexamples in justifications.

**Debbie**: OK, so Jason, do we have some justification either that this statement [An exterior angle is always obtuse] is true or that this statement is false? If so, what is our supporting argument? [She points to “Use examples and counterexamples” on the poster.] We’re looking for examples to either support this or a counterexample to show that this is false. Adam?

**Adam**: Well, justification as false is that we tried it on a real-life angle. We tried to measure an angle, and it was 33.6 degrees.

**Debbie**: OK. [Writing in the justification space on the second poster: “This is false. We are able to make an exterior angle with measure 33.6.”]

This excerpt shows how Debbie’s students, designated special education and/or English language learners, were able to use counterexamples. They also received guidance from their teacher, including a description of a counterexample. Debbie then further clarified and labeled the use of counterexample for the class by saying, “This example where we found an exterior angle that had a measure that was acute and less than its adjacent interior angle—this is what we call a counterexample.” She then explained that “it shows that our conjecture is false.”

Her words elaborated on the poster’s advice and summarized students’ discussion by explaining what a counterexample is and how it is used. Evidence of her effectiveness is found in the gains that students made on a pretest and posttest of content of the Triangles unit (comparable to the gains of middle- and high-achieving students in other classes).

Building off other people’s ideas is the basis for the social process of argumentation. It is possible to have a lot of individual justifications, given one after the other, and label that as a classroom discussion, but then students would be missing out on the opportunity to understand the mathematical ideas of others.

Stephanie, using the Patterns of Coordinates activities, helped students build off others’ ideas. In the midst of
an activity using rectangles, she realized that students needed a better definition of rectangles. Making good definitions is a special kind of argumentation. In this situation, the proposed definition is a conjecture. The justification entails whether the definition includes all cases of the mathematical concept being defined and excludes all cases that are nonexamples. Stephanie explained to students:

We’re going to establish the definition of a rectangle. Please make sure you’re paying attention to each other’s comments. Remember, I want you to speak even if what you want to say is a question. I want to see you participating, so that you can get credit for math argumentation.

One student, Ethan, stated that a rectangle is a four-sided figure with two sets of different sides with four right angles. Darian, another student, defined a rectangle as a figure with two sets of parallel and two sets of vertical sides. Ethan and Darian each gave a definition of a rectangle, but Darian’s thoughts seemed independent of Ethan’s. Stephanie further explained how to “build off each other’s ideas”: During math argumentation, students should respond to each other’s comments, rather than simply make individual statements. Most students need reminders of and encouragement about correct behavior when making arguments—or in this case, definitions—as a whole class.

The class continued, as Olivia walked to the board and noted—

A square is not a rectangle. Square is going be like that [draws a square] and rectangle isn’t, like that, and they don’t have to be equal [draws a rectangle, vertically elongated].

Olivia’s comment is related to Ethan’s and Darian’s but she did not make an explicit connection. She offered the conjecture that a square is not a rectangle, rather than a definition of a rectangle. Another student, Matthew, agreed with Ethan’s statement and expressed that a rectangle can be a square but a square cannot be a rectangle.

Although Matthew said he was responding to Ethan, his statement in fact is related to Olivia’s. Stephanie passed the baton to Jacob, who took a more summary approach and addressed all the previous attempts at definition as he walked to the board:

OK. I disagree with what Matthew, Darien, Kel, and Ethan said because uh—a rectangle is a quadrilateral with all straight—OK. A closed figure with all straight lines with four 90 degree angles and two sets of parallel sides.

Jacob did a better job of connecting his definition of a rectangle to what other students had said. The class built on this definition to conclude that a square is a rectangle, but that not all rectangles are squares.

Students did not immediately follow Stephanie’s directive to build off each other’s statements. Instead, a gradual shift was made toward relating new ideas to what students had already said. Stephanie told students what was expected of them and managed their turn taking. Students’ contributions were mathematically meaningful and lengthy enough to portray complex ideas. Stephanie’s moves demonstrate that setting clear expectations through telling students how they should behave during argumentation gradually pays off.

In argumentation, students try to convince others of their opinions. However, teachers need to keep in mind that students could be wrong, which is acceptable. This norm can be difficult to support. Teachers can remind students that conjectures are not considered true until there is a justification that satisfies everyone. On the other hand, students may want to “disown” conjectures that turn out to be false.

Bernie was working with a pair of students on the Triangles activities, in front of the whole class. They had posed the conjecture that an exterior angle of a triangle is always larger than the interior. Several pairs of students around the room started making counterexamples using dynamic geometry software. But Bernie let the students who made the conjecture justify that it was false. She told them that even if they find their conjecture to be false, it is still a justification and provides a counterexample.

Students can feel uncomfortable being associated with false statements. Making a conjecture that is judged as false or wrong may go against the grain of what they have learned about being in mathematics class. Bernie worked to overcome this feeling by praising the conjecture that turned out to be false for its role in developing argumentation.

Concluding

Concluding is, in essence, a time for class agreement that a conjecture is true or false, assuming that an argument has been persuasive and correct. Bernie ended each class period with concluding. With the class, she examined each argument that had been made for a conjecture and “stamped” them as true or false. She prompted students to add the true statements to their notebook.

Bernie asked if everyone agreed. In cases where they did not, she continued the argument until all were convinced. She had them add this proven conjecture, a fact, to a list they had made in their workbooks. Both stamping and adding to a list are concrete aspects of concluding.
SETTING EXPPLICIT EXPECTATIONS

All four teachers in the vignettes above succeeded in supporting argumentation by students over several class periods—far more than we can share in this article. These teachers were explicit about what was expected of students and what made for good argumentation. As a community of mathematics educators, we know that telling is not all there is to teaching, but these examples show us that telling has its place in setting expectations for what constitutes mathematical argumentation. Additionally, what we tell students matters: Advice to help make argumentation more accessible by indicating specific behaviors and norms can make a difference in the amount and quality of students’ participation in argumentation.

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CCSSM Practices in Action

SMP 3: Construct viable arguments and critique the reasoning of others.
SMP 5: Use appropriate tools strategically.

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Any thoughts on this article? Send an e-mail to mtms@nctm.org.—Ed.

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