Create homework assignments that both engage middle-grades students and strengthen their mathematical understandings and skills.

Parents in the United States expect their students to have homework; and students, especially in middle school and high school, expect daily homework assignments from their teachers. In fact, the authors of The Metlife Survey of the American Teacher (Metlife Foundation 2007) reported that 90 percent of surveyed teachers agree with the following statement: Doing homework helps students learn more in school. Furthermore, in the same study, 50 percent of students in grades 7–12 reported typically having one or more hours of homework on a weeknight; 8 percent reported being assigned three or more hours of homework.

In Rethinking Homework: Best Practices That Support Diverse Needs, Cathy Vatterott notes that the practice of assigning homework began when schools were dominated by rote learning (Vatterott 2009). Middle-grades teachers are now incorporating more sense-making activities into their classrooms (many of which can be found in Mathematics Teaching for the Middle School). We contend that students can also engage in sense-making activities outside the classroom and that homework can consist of more than just “tasks of memorization and practice” that Vatterott describes. We believe that completing homework can help middle school students strengthen mathematical understandings and skills.

However, it is difficult to create effective homework assignments. In thinking back to our own time as math teachers, we realized that many of our assignments were often “too easy” for some students, “too hard” for others, and “just right” for only a few. Additionally, as you have probably experienced, we had students who did not complete homework assignments or did so with little mathematical thinking. It is no wonder that math teachers...
often say that “homework” is at the top of their list of concerns.

Despite the challenges involved, we believe that homework can be an important component of mathematics learning. If you have found yourself agreeing with both the importance of homework and the challenges that homework entails, then perhaps it is time for you to integrate new, potentially more effective homework practices into your math instruction.

In this article, we first describe two important ideas about homework. We hope that these ideas will help you begin to rethink the purposes of homework and support you in making effective choices about assigning homework. We then describe specific strategies for choosing, revising, or creating assignments that help students learn from homework. Over time, keeping these big ideas in mind and following some of these suggestions can help make homework more productive for you and your students.

TWO BIG IDEAS ABOUT HOMEWORK

We have found that discussions and decisions about homework are greatly simplified by keeping two big ideas in mind (see fig. 1).

1. Effective homework supports mathematical learning by including tasks that require students to think about important mathematics. The idea that students should think about important math while doing homework seems fairly obvious. However, if we stop to examine how our students often experience homework, we might discover that many of them do not engage in much mathematical thinking. All too often, students work to complete their homework as quickly and with as little thought as possible. Perhaps the assignment is too easy, in which case students are completing problems that do not require much thought. Perhaps the assignment is too difficult, and students simply have no way to get started. In both cases, engaging with homework is not helping students learn.

   For students to learn while working on mathematical problems, they must actively engage in mathematical thinking. This means that if we want students to learn mathematics (e.g., better understand mathematical relationships, become more proficient at mathematical procedures, and become better problem solvers) while doing homework, we must design tasks that require them to solve problems, reason, represent, communicate, and make connections (NCTM 2000).

2. Teachers must be clear about goals and expectations. Again, it may appear that we are simply stating the obvious.
However, in our experience, we have found that often our students do not share with us the same assumptions about the purposes of homework or how they are expected to engage with it. For instance, we expect that students may struggle with solving some of the problems, and we expect that they may need to refer to their notes or ask a friend for help. However, our students may think that they are supposed to be able to complete all the homework problems quickly and easily, and that referring to their notes or asking a friend for help is somehow “cheating.”

We think that this disconnect comes from our lack of clarity. The clearer we are with our students about our goals for homework and how we expect them to work on it, the more likely that everyone is on the same page with regard to how to learn mathematics while doing homework.

PRACTICAL SUGGESTIONS

Having these big ideas in mind helps focus thinking about homework, but these big ideas are not enough. In the following subsections, we offer suggestions for modifying existing homework assignments to increase students’ active thinking.

We propose (and then expand on) the following three ways that you can use to enhance students’ opportunities to learn math while doing homework:

1. Create tasks that require deep thinking
2. Encourage metacognition
3. Teach explicit strategies for success

Create Tasks That Require Deep Thinking

For students to learn from doing homework, they need problems and questions that require them to think, to solve problems, to reason, to represent, to communicate, and to make connections. Moreover, these processes must be integrated throughout the homework assignment. You can use the following strategies for designing homework assignments whose completion requires students to think about mathematics. Imagine that your textbook contained the homework problems shown in figure 2. How might you modify this homework to align it with the big ideas we have described?

Give students “model solutions” to correct or criticize. When you give students a variety of model answers, some that are wholly correct, some that contain minor errors, and others that represent major misconceptions, they will have to differentiate between conceptual errors and procedural ones as well as work to understand the reasoning of others. These kinds of tasks
force students to be thoughtful and engage at a deeper level than simply practicing learned procedures. See figure 3, which contains three different answers that students could critique from the problems listed in figure 2.

Have students sort problems. When students sort a variety of problems (or solutions) into categories, they make connections among problems, representations, and larger mathematical concepts. Using the problems from figure 2 as a starting point, you could have students sort the six problems into categories of their choosing. Students might sort problems in this way: One category can contain problems involving area formulas of circles; another category can include area formulas for rectangles; and yet another category can comprise a combination of different area formulas. Other students might group together (1) the problems that require adding areas and (2) the problems that require subtracting areas. An important component of this type of activity is for students to communicate, in writing, the criteria they used to sort the problems. You can establish the criteria for the categories, or you can have students create their own criteria as part of the assignment.

Have students create problems with particular features. Asking students to create their own problems is a great way to get them to think about the underlying structure of the mathematics they are learning. Students working on the homework from figure 2 could create a problem that involves a circle and a rectangle or that requires adding areas. They could also design a problem that involves subtracting a circle from a trapezoid or that combines two shapes to create a new shape with an area of 24 square feet.

No matter which of these strategies you use, we recommend that you give problems that require deeper thinking at the beginning of the assignment, and then encourage students to continue thinking about them as they complete other homework problems. They may need to come back to those “thinking” problems after having worked through other tasks that are more accessible. For instance, if at first they cannot make much sense of a homework prompt that requires them to write a problem that involves subtracting areas, they will better

<table>
<thead>
<tr>
<th>Problem</th>
<th>Student Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. In each problem below, find the area of the shaded region.</td>
<td>16 × 10 = 160 4 × 4 = 16 160 + 16 = 176 Answer: 176 square feet</td>
</tr>
<tr>
<td>2.</td>
<td>7 × 4 = 28 3.14 × 4² = 3.14 × 16 = 50.24 50.24 ÷ 2 = 25.12 25.12 + 28 = 53.12 Answer: 53.12 cm</td>
</tr>
<tr>
<td>3.</td>
<td>25 × .5 + 25 × 3.14 25 × 3.64 = 91 Answer: 91 square meters</td>
</tr>
</tbody>
</table>

Fig. 3 These model solutions allow students to critique the final answers.
Encourage Metacognition

Metacognition is being aware of one’s own thinking. Research in cognitive science has found that experts are effective at this kind of mental monitoring. When experts solve problems, they constantly assess if they are making progress toward a valid solution. When experts are presented with new information or ideas, they stop to consider whether they are making sense of it, and they employ strategies, such as formulating a question, if they find they are not understanding (NRC 2000).

Successful math students do this as a matter of course. However, there are many students who do not. Clearly, we want students to engage thoughtfully with their math homework; we also want them to pay attention to what they are doing and monitor whether what they are doing is working. You can use the following strategies to encourage metacognition.

Have students explain their thinking before practicing procedures. Many textbooks contain problems that ask students to explain how to do a procedure at the end of a series of exercises. When designing homework, you can instead place such questions at the beginning of an assignment and ask students to revise the explanations as they work, thus requiring them to practice the procedures. In this way, you can have all students engage in the kind of thinking that many high-achieving students engage in intuitively—thinking about how they are doing the procedure in general terms and getting more and more clear about what they are doing over time.

Have students evaluate their performance and indicate problem difficulty. Another way to foster metacognition is to have students indicate how confident they are of their answers to homework problems. When students develop the ability to pinpoint exactly what is difficult for them or what they do not understand, they are taking their first step in moving toward understanding.

You can have students indicate, next to each problem, if they think the solution is correct (C) or wrong (W) or if they are unsure (U). Some teachers may prefer to use a scale of 1–5, with 5 indicating high confidence in correctness and 1 indicating low confidence. You could also have students indicate how difficult they found the problem, which is similar to having them indicate if they think they got it right or wrong but not exactly the same. Some problems may be difficult for students, but they still may be confident that they found the correct solution.

Have students describe why they are “stuck.” A strategy that we have found particularly helpful in getting students to think about their own mathematical thinking is to encourage them to describe why they are stuck on a particular problem. The mere act of “diagnosing” where the difficulty lies can help a student figure out what to do next. Consider giving credit on homework when students can explain why they are stuck, even when they cannot give a complete solution to the problem. If you choose this strategy, then it will be important that you teach your students what kind of “I’m stuck because” response counts for full credit.

Have students explain what strategies they use to get “unstuck.” Students often think that they are supposed to know exactly how to solve a math problem when they first see it. Asking students to describe what they did when they got stuck on a homework problem reinforces your expectation that they will not always know what to do right away. Writing down what they have tried, including going down “wrong” solution paths, will help them move from a sense of helplessness when they do not know immediately what to do to a sense of confidence in the face of initial difficulties.

Teach Explicit Strategies for Success

Students who successfully learn by completing homework use strategies that all students can learn to use. You can use the following moves when teaching your students effective homework strategies.

Name successful strategies. Before you can teach strategies, you need to be able to name and describe them. The sidebar on page 165 lists a number of strategies that we have identified through our own teaching. You may know of others to add to the list. We have found it helpful to have such a list posted prominently in our classrooms.
Model and practice homework strategies. In addition to naming these strategies, take time during class to model the strategies and have students practice them while you observe and give feedback. Show your students how you would do a homework assignment, naming and demonstrating each strategy as you go. Then have them do homework in class. Circulate around the room and give explicit positive feedback when students use the strategies you have described. If students ask for help, ask whether they have used any of the strategies you have modeled. If they cannot remember the strategies, refer them to the poster with the named strategies.

Assign Homework Buddies
It may not be enough to tell students to ask for help when they run into difficulties while doing homework. It is also helpful to assign each student a homework buddy who can be contacted when a question comes up. When a student knows that he or she has “permission” to call a designated classmate when trouble arises, it allows the student to internalize that questions are acceptable. In addition, the buddy can also provide an avenue for needed help. Be sure that homework buddies communicate when it is appropriate to call and negotiate how long calls can last. When you first assign homework buddies, have classmates contact their buddies as part of the assignment.

Middle-grades students can and should be learning mathematics through engaging in sense-making activities in class and outside class. Just as we want students to engage in learning during math class through problem solving, connecting, representing, communicating, and reasoning (NCTM 2000), we also want our students to do their homework and learn from the experience. If we are clear about creating homework assignments whose completion explicitly requires students to think about mathematics and to pay attention to that thinking, and if we are clear with our students about how we expect them to engage with their homework, then they will be much more likely to learn from doing homework and develop important mathematical habits of thinking. The strategies we have described in this article can help you establish effective homework practices for both you and your students.

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REFERENCES

Any thoughts on this article? Send an email to mtms@nctm.org.—Ed.